平成27年度 情報工学コース卒業研究報告要旨

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卒業研究題目	An extension of proof graphs for disjunctive parameterised Boolean equation systems	

A Parameterised Boolean Equation System (PBES) is a set of equations that defines sets as the least and/or greatest fixed-points that satisfy the equations. PBESs are used for solving variety problems such as process equivalences, model checking problems, and so on. The membership problem of PBESs is the problem deciding whether v is in X for a data v and a set X defined in a PBES. The membership problem is undecidable in general, and hence some techniques have been proposed that solve the problem for some subclass of PBESs. The proof graph scheme is one of the techniques. In this method, the membership problem is reduced to the existence of a proof graph. A mechanical search of a proof graph is easy if there exists finite one. In general, proof graphs are infinite, and it is difficult to find a proof graph. Therefore, a method is required that solve the membership problem on a PBES which has no finite proof graphs.

This paper studies a technique to solve the membership problem on a subclass of disjunctive PBESs, by extending the notion of proof graphs. Each vertex $X(v)$ in a proof graph is feasible, i.e., the data v is in the set X, if the graph satisfies conditions induced from a given disjunctive PBES. An important condition among them is that the feasibility of each vertex is induced from the assumption that all vertices in its post set are feasible. For a disjunctive PBES, it is shown that each vertex has at most one edge. For this reason, each edge in a proof graph represents that the vertex is feasible if the terminal vertex is. In other words, each edge represents the dependency of the membership problem on a given disjunctive PBES. Since proof graphs are possibly infinite, we extend proof graphs by introducing vertices each of which stands for a set of vertices of the original ones. In this graph, each vertex is written as $X(A)$ for some set A, and an edge from $X(A)$ to $X(B)$ means that all vertices in $X(A)$ are feasible if all vertices in $X(B)$ are. We show that the existence of an extended proof graph for a given PBES coincides with the existence of a proof graph. Therefore, extended proof graphs really extend ones. We also show that examples of a disjunctive PBES having no finite proof graph except for extended one.

We also propose a graph, called a dependency space, to construct an extended proof graph. Dependency spaces represent the relation among the membership problems on a given disjunctive PBES. Each vertex in a dependency space is a set of vertices in a proof graph, and all vertices in each set have the same dependency. There exists an edge between two vertices if the initial vertex has a dependency on the terminal vertex. We define the dependency spaces by the congruence relation on many-sorted algebra whose sorts are corresponded to the predicate variables. We show that a dependency space contains an extended proof graphs in its sub-graphs if there exists a proof graph. Thereby, we can check the existence of an extended proof graph by constructing a dependency space. We also show that examples of a disjunctive PBES that induce a finite dependency space, which contains an extended proof graph.